UNIT 4 TRANSFORMER

Structure

4.1 INTRODUCTION

The transformer is a device that transfers electrical energy from one electrical circuit to another electrical circuit. The two circuits may be operating at different voltage levels but always work at the same frequency. Basically transformer is an electro-magnetic energy conversion device. It is commonly used in electrical power system and distribution systems.

In this unit, we will first get an understanding of the physical principle of operation and construction of transformer. Thereafter, we will study in detail the operation of transformer at load.

In particular, we will consider the representation of the transformer using equivalent circuits for estimating voltage and efficiency at various loads. Apart from ac power system, transformers are used for communication, instrumentation and control. In this unit, you will be introduced to the salient features of instrument transformers.

This unit ends by considering the use of three phase transformers, and basics of thee phase bank of single-phase transformers.

Objectives

After studying this unit, you should be able to

- explain the voltage and current converting capability of the transformer,
- understand equivalent circuit representation of transformer and prediction of voltage regulation and efficiency at different loads,
- determine the equivalent circuit parameters by conducting simple tests,
- understand operation of auto-transformers, instrument transformers, and
- understand transformers used in three phase systems.

4.2 BASICS OF TRANSFORMER

4.2.1 Introduction

In its simplest form a single-phase transformer consists of two windings, wound on an iron core one of the windings is connected to an ac source of supply *f*. The source supplies a current to this winding (called primary winding) which in turn produces a flux in the iron core. This flux is alternating in nature (Refer Figure 4.1). If the supplied voltage has a frequency *f*, the flux in the core also alternates at a frequency *f*. the alternating flux linking with the second winding, induces a voltage E_2 in the second winding (called secondary winding). [Note that this alternating flux linking with primary winding will also induce a voltage in the primary winding, denoted as E_1. Applied voltage V_1 is very nearly equal to E_1]. If the number of turns in the primary and secondary windings is N_1 and N_2 respectively, we shall see later in this unit that $\frac{E_1}{E_2} = \frac{N_1}{N_2}$. The load is connected across the secondary winding, between the terminals a_1, a_2. Thus, the load can be supplied at a voltage higher or lower than the supply voltage, depending upon the ratio $\frac{N_1}{N_2}$.

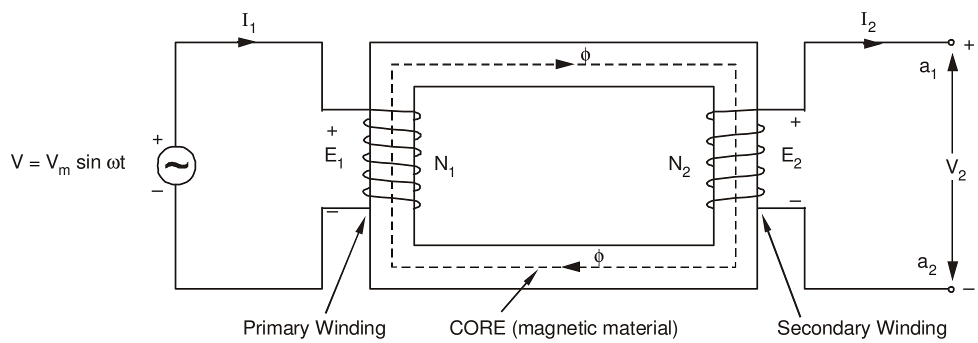


Figure 4.1 : Basic Arrangement of Transformer

When a load is connected across the secondary winding it carries a current I_2, called load current. The primary current correspondingly increases to provide for the load current, in addition to the small no load current. The transfer of power from the primary side (or source) to the secondary side (or load) is through the mutual flux and core. There is no direct electrical connection between the primary and secondary sides.

In an actual transformer, when the iron core carries alternating flux, there is a power loss in the core called core loss, iron loss or no load loss. Further, the primary and secondary windings have a resistance, and the currents in primary and secondary windings give rise to I^2R losses in transformer windings, also called copper losses. The losses lead to production of heat in the transformers, and a consequent temperature rise. Therefore, in transformer, cooling methods are adopted to ensure that the temperature remains within limit so that no damage is done to windings' insulation and material.

4.2.2 EMF Equation of a Transformer

In the Figure 4.1 of a single-phase transformer, the primary winding has been shown connected to a source of constant sinusoidal voltage of frequency f Hz and the secondary terminals are kept open.

The primary winding of N_1 turns draws a small amount of alternating current of instantaneous value i_0, called the exciting current. This current establishes flux ϕ in the core (+ve direction marked on diagram). The strong coupling enables all of the flux ϕ to be confined to the core (i.e. there is no leakage of flux). Consequently, the flux linkage of primary winding is

$$\lambda_1 = N_1 \phi \qquad \ldots (4.1)$$

and the flux linkage λ_2 of the secondary winding is

$$\lambda_2 = N_2 \phi \qquad \ldots (4.2)$$

The time rate of change of these flux linkages induces emf in the windings given by

$$e_1 = \frac{d\lambda_1}{dt} = - N_1 \frac{d\phi}{dt} \qquad \ldots (4.3)$$

and

$$e_2 = \frac{d\lambda_2}{dt} = - N_2 \frac{d\phi}{dt} \qquad \ldots (4.4)$$

As per Lenz's law, the positive direction of the induced emf opposes the positive current direction and is shown by (+) and (–) polarity marked on the diagram.

Assuming the ideal case of the windings possessing zero resistance, as per KVL, we can write

$$v_1 = e_1 \qquad \ldots (4.5)$$

Thus, both e_1 and ϕ must be sinusoidal of frequency f Hz, the same as that of the voltage source. (Consequently, e_2 is also of same frequency and hence the definition of transformer should incorporate the "same frequency" concept).

Let

$$\phi = \phi_m \sin \omega t \qquad \ldots (4.5a)$$

Where, $\omega = 2\,\pi f$, and ϕ_m is the peak (maximum) value of the flux.

From Eq. (4.3),

$$\begin{aligned} e_1 &= - N_1 \frac{d\phi}{dt} \\ &= - N_1 \,.\, \phi_m \,.\, \omega \cos \omega t \qquad \ldots (4.6) \\ &= (\omega\, N_1\, \phi_m) \sin \left[\omega t + \frac{\pi}{2}\right] \end{aligned}$$

$$e_1 = E_{m1} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots (4.6a)$$

where, $E_{m1} = \omega\, N_1\, \phi_m$

From Eq. (4.4)

Similarly,

$$e_2 = E_{m2} \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \ldots (4.6b)$$

where, $E_{m2} = \omega\, N_2\, \phi_m$

Eqs. (4.6a) and (4.6b) indicate that both E_1 and E_2 lag ϕ (Eq. (4.5a)) by 90°.

RMS Value of Induced emf

The RMS values of the induced emf in the primary and secondary windings, E_1, E_2 are given by

$$E_1 = \frac{E_{m1}}{\sqrt{2}} \text{ and } E_2 = \frac{E_{m2}}{\sqrt{2}}$$

or,
$$E_1 = \frac{1}{\sqrt{2}} \,.\, \omega N_1 \phi_m$$

$$= \frac{1}{\sqrt{2}} \,.\, 2\pi f N_1 \phi_m$$

$$= \sqrt{2}\, \pi f N_1 \phi_m$$

$$= 4.44 f N_1 \phi_m \qquad \ldots (4.7)$$

Similarly, $E_2 = 4.44 f N_2 \phi_m$ . . . (4.8)

Dividing Eqs. (4.7) by (4.8)

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k. \qquad \ldots (4.9)$$

The turns ratio is denoted by 'k' and has no unit as it is a ratio.

If $k < 1$, the secondary-voltage is less than the primary voltage and the transformer is called a step-down transformer. If $k > 1$, secondary voltage is more than the primary voltage (step up transformer).

Example 4.1

A single-phase transformer has 500 primary and 1000 secondary turns. The net cross-sectional area of core is 60 cm^2. If the primary winding be connected to 50 Hz supply at 400 V, calculate (a) the peak value of flux density in core, (b) the voltage induced in the secondary winding and
(c) the turns ratio.

Solution

Primary induced emf $E_1 = V_1 = 400$ V

Supply frequency $f = 50$ Hz

No. of turns in primary winding = 500

Area of core = 60 cm^2 = 0.006 m^2

We know that

$$E_1 = 4.44\, \phi_{max} f N_1$$

So
$$\phi_{max} = \frac{E_1}{4.44 f N_1}$$

$$= \frac{400}{4.44 \times 50 \times 500} = 3.6 \times 10^{-3} \text{ Webers}$$

(a) Peak value of flux density

$$B_{max} = \frac{\phi_{max}}{\text{area } (a)} = \frac{3.6 \times 10^{-3}}{0.006 \text{ m}^2}$$

$$= \frac{0.0036}{0.006} = 0.6 \text{ Tesla (Weber/m}^2)$$

(b) Turns ratio $\frac{N_2}{N_1} = \frac{1000}{500} = \frac{2}{1}$

(c) Induced voltage in secondary winding

$$E_2 = E_1 \text{ . Turns Ratio}$$

$$= 400 \times 2 = 800 \text{ Volt} .$$

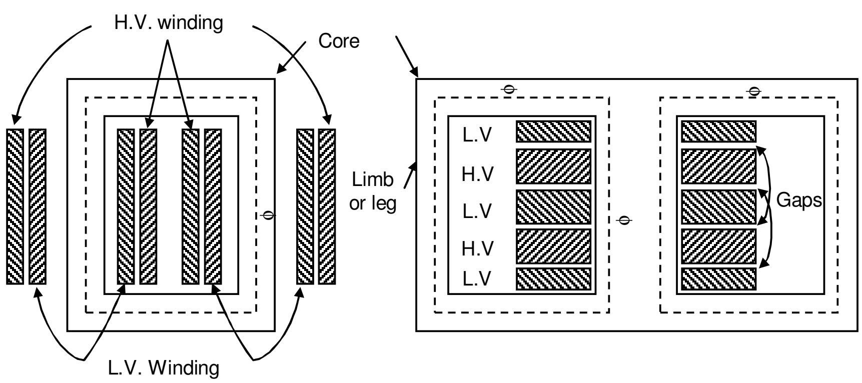
SAQ 1

A single phase transformer has a core, whose cross-sectional area is 150 cm^2, operates at a maximum flux density of 1.1 Wb/m^2 from 50 Hz supply. The secondary winding has 66 turns. Determine output in kVA when connected to a load of 4 Ω impedance. Assume all voltage drops to be negligible.

4.2.3 Construction

Core-type and Shell-type Construction

Depending upon the manner in which the primary and secondary windings are placed on the core, and the shape of the core, there are two types of transformers, called (a) core type, and (b) shell type. In core type transformers, the windings are placed in the form of concentric cylindrical coils placed around the vertical limbs of the core. The low-voltage (LV) as well as the high-voltage (HV) winding are made in two halves, and placed on the two limbs of core. The LV winding is placed next to the core for economy in insulation cost. Figure 4.2(a) shows the cross-section of the arrangement. In the shell type transformer, the primary and secondary windings are wound over the central limb of a three-limb core as shown in Figure 4.2(b). The HV and LV windings are split into a number of sections, and the sections are interleaved or sandwiched i.e. the sections of the HV and LV windings are placed alternately.



(a) Core Type **(b) Shell Type**

Figure 4.2 : Windings and Core in Core Type and Shell-Type Transformer

Core

The core is built-up of thin steel laminations insulated from each other. This helps in reducing the eddy current losses in the core, and also helps in construction of the transformer.

The steel used for core is of high silicon content, sometimes heat treated to produce a high permeability and low hysterisis loss. The material commonly used for core is CRGO (Cold Rolled Grain Oriented) steel.

Conductor material used for windings is mostly copper. However, for small distribution transformer aluminium is also sometimes used. The conductors, core and whole windings are insulated using various insulating materials depending upon the voltage.

Insulating Oil

In oil-immersed transformer, the iron core together with windings is immersed in insulating oil. The insulating oil provides better insulation, protects insulation from moisture and transfers the heat produced in core and windings to the atmosphere. The transformer oil should posses the following quantities :

(a) High dielectric strength,

(b) Low viscosity and high purity,

(c) High flash point, and

(d) Free from sludge.

Transformer oil is generally a mineral oil obtained by fractional distillation of crude oil.

Tank and Conservator

The transformer tank contains core wound with windings and the insulating oil. In large transformers small expansion tank is also connected with main tank is known as conservator. Conservator provides space when insulating oil expands due to heating. The transformer tank is provided with tubes on the outside, to permits circulation of oil, which aides in cooling. Some additional devices like breather and Buchholz relay are connected with main tank.

Buchholz relay is placed between main tank and conservator. It protect the transformer under extreme heating of transformer winding. Breather protects the insulating oil from moisture when the cool transformer sucks air inside. The silica gel filled breather absorbes moisture when air enters the tank. Some other necessary parts are connected with main tank like, Bushings, Cable Boxes, Temperature gauge, Oil gauge, Tappings, etc.

4.3 EQUIVALENT CIRCUIT OF TRANSFORMER

The performance of a transformer at no load and at load is influenced by mutual flux, the leakage fluxes, the winding resistances and the iron losses. For the purpose of performance evaluation, the effect of these is represented on an electrical circuit, in the form of resistances and reactances. Such an electrical circuit is called "equivalent circuit."

In this section, we will develop the equivalent circuit of a single-phase transformer in the following steps :

(a) Equivalent circuit of an ideal transformer at no load

(b) Equivalent circuit of an ideal transformer on load

(c) Equivalent circuit at load

(d) Equivalent circuit referred to primary side

(e) Approximate equivalent circuit.

4.3.1 Equivalent Circuit of an Ideal Transformer at No Load

Under certain conditions, the transformer can be treated as an ideal transformer. The assumptions necessary to treat it as an ideal transformer are :

(a) Primary and secondary windings have zero resistance. This means that ohmic loss ($I^2 R$ loss), and resistive voltage drops in windings are zero.

(b) There is no leakage flux, i.e. the entire flux is mutual flux that links both the primary and secondary windings.

(c) Permeability of the core is infinite this means that the magnetizing current needed for establishing the flux is zero.

(d) Core loss (hysteresis as well as eddy current losses) are zero.

We have earlier learnt that :

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = k$$

(k is a constant, known as voltage transformation ratio or turns ratio).

For an ideal transformer, $V_1 = E_1$ and $E_2 = V_2$.

$$\therefore \qquad \frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Even at no load, a transformer draws some active power from the source to provide the following losses in the core :

(a) eddy-current loss, and

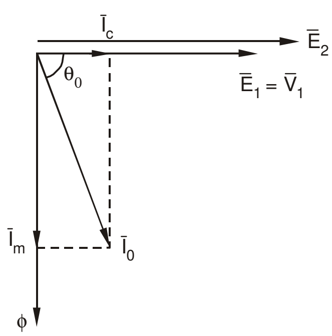
(b) hysteresis loss.

The current responsible for the active power is nearly in phase with V_1 (applied voltage) and is known as core-loss current. A transformer when connected to supply, draws a current to produce the flux in the core. At no-load, this flux lags nearly by 90° behind the applied voltage V_1. The magnetizing current, denoted by I_m is in phase with the flux ϕ and thus, lags behind the applied voltage by nearly 90°. The phasor sum of the core loss component of current I_c and the magnetizing current I_m is equal to the no-load current $\bar{I}_0$.

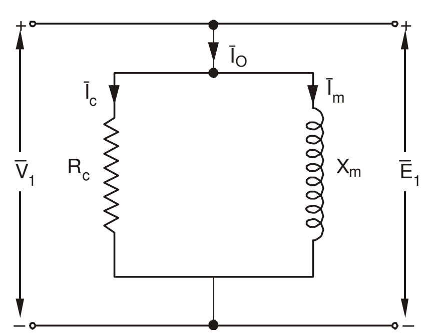
$$I_c = I_0 \cos \phi_0 \text{ and } I_m = I_0 \sin \phi_0$$

$$\text{Core loss} = P_0 = V_1 I_0 (\cos \phi_0)$$

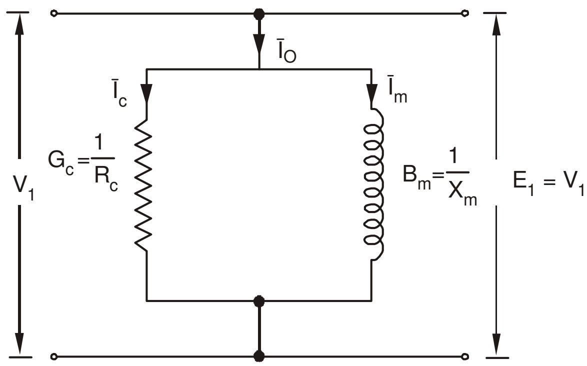
where ϕ_0 is the phase angle between V_1 and I_0, and, $(\cos \phi_0)$ is the no load power factor. The phase relationship between applied voltage V_1, no-load current I_0, and its components I_c, I_m is shown in Figure 4.3(a).



(a) Phasor Diagram at No Load



(b) Equivalent Circuit at No Load



(c) Equivalent Circuit Alternative Representation

Figure 4.3

In the form of equivalent circuit, this can be represented as Figure 4.3(b), in which R_c is a resistance representing core loss and X_m is an inductive reactance (called magnetizing reactance). Note that the current in the resistance is in phase with V_1 and X_m being an inductive reactance, the current I_m in this branch lags V_1 by 90° as shown in the phasor diagram of Figure 4.3(a).

(The representation in Figure 4.3, assumes that $V_1 = E_1$ (equal to and in opposition to V_1). This implies that the primary winding resistance and leakage reactance are neglected. Similarly, in the secondary winding of transformer mutually induced emf is antiphase with V_1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns. (You will learn about the concept of leakage reactance when you study about the equivalent circuit at load).

The equivalent circuit parameters R_c and X_m can also be expressed as conductance and susceptance G_c, B_m such that

$$R_c = \frac{V_1^2}{P_0}, \quad I_c = \frac{V}{R_c}, \quad P_0 = I_c^2 \, R_c$$

Also,
$$X_m = \frac{V_1}{I_m} \text{ or } I_m = \frac{V_1}{X_m}$$

Example 4.2

At no-load a transformer has a no-load loss of 50 W, draws a current of 2A (RMS) and has an applied voltage of 230V (RMS). Determine the (i) no-load power factor, (ii) core loss current, and (iii) magnetizing current. Also, calculate the no-load circuit parameter (R_c, X_m) of the transformer.

Solution

$$P_c = 40 \text{ W}$$

$$I_0 = 2 \text{ A}$$

$$E_1 = 230 \text{ V}$$

$$P_c = V_1 \, I_0 \cos \phi_0$$

$$\Rightarrow \quad \cos \phi_0 = \frac{P_c}{E_1 \, I_0}$$

$$= \frac{50}{230 \times 2} = 0.108 \text{ lagging}$$

$$\Rightarrow \quad \phi_0 = 83.76^\circ$$

Magnetizing current, $I_m = I_0 \sin \phi_0$

$= 2 \sin (83.76^{o})$

$= 1.988$ A

Core-loss current $I_c = I_0 \cos \phi_0$

$= 2 \times 0.108$

$= 0.216$ A

$$P_c = \frac{V_1^2}{R_c} = G_c \cdot V_1^2 \Rightarrow R_c = \frac{V_1^2}{P_c} = \frac{230^2}{50} = 1.058 \text{ k}\Omega$$

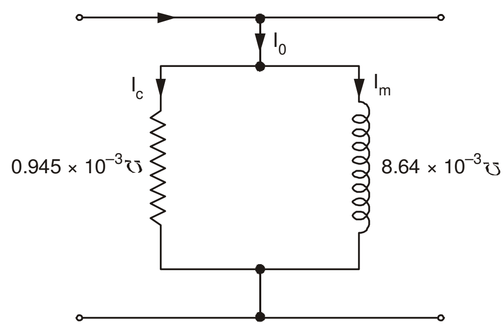
$$G_c = \frac{50}{(230)^2} = 0.945 \times 10^{-3} \text{ (mho)}$$

Also, $$I_m = B_m \cdot V_1 = \frac{V_1}{X_m}$$

$$\Rightarrow \quad X_m = \frac{V_1}{I_m} = \frac{230}{1.988} = 115.69 \ \Omega$$

$$\Rightarrow \quad B_m = \frac{I_m}{V_1} = \frac{1.988}{230} = 8.64 \times 10^{-3} \text{ (mho)}$$

This equivalent circuit is shown below.



Figure

4.3.2 Equivalent Circuit of an Ideal Transformer on Load

Under certain conditions the transformer can be treated as an ideal transformer. The idealizing assumptions are listed below :

(a) Both primary and secondary windings have zero resistance. This means, no ohmic power loss and no resistive voltage drop.

(b) No leakage flux, i.e. all the flux produced is confined to the core and links both the windings

(c) Infinite permeability of the core. This means no zero magnetizing current is needed to establish the requisite amount of flux in the core, i.e. $I_m = 0$.

(d) Core-loss (hysteresis as well as eddy-current loss) is zero, i.e. $I_c = 0$. Assumptions (a), (b) and (d) mean that copper losses, and iron losses being zero, the efficiency of the transformer is 100%. Since $I_m = I_c = 0$, $I_0 = 0$.

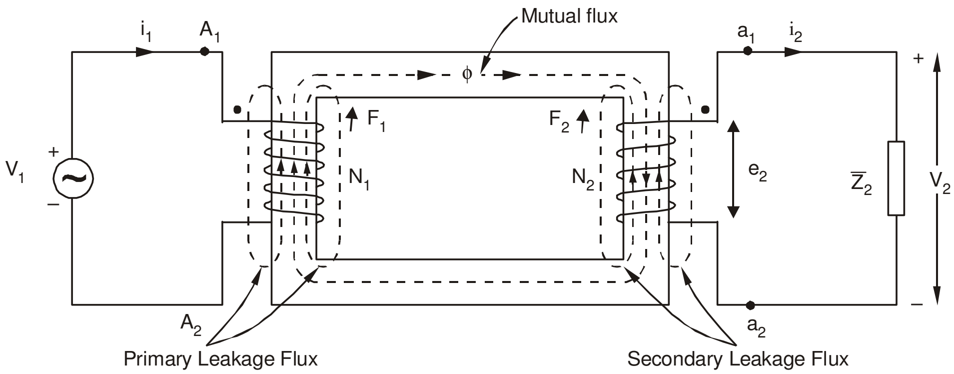


Figure 4.4 : Transformer on Load

As per earlier derived equation

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

where, V_1 is supply voltage and V_2 is voltage across load terminals.

When load is applied, let the impedance of load be Z_L, as shown in Figure 4.4.

Sinusoidal current i_2 flows through the secondary.

Therefore, secondary winding creates an mmf $F_2 = N_2\, i_2$ which opposes the flux ϕ.

But mutual flux ϕ is invariant with respect to load (otherwise $v_1 = e_1$ balance is disturbed).

As a result, the primary winding starts drawing a current i_1 from the source so as to create mmf $F_1 = N_1\, i_1$ which at all times cancels out the load-caused mmf $N_2\, i_2$ so that ϕ is maintained constant independent of the magnitude of the load current which flows in the secondary winding. This implies that for higher load, more power will be drawn from the supply.

Thus, $$N_1\, i_1 = N_2\, i_2 \Rightarrow \frac{N_2}{N_1} = \frac{i_1}{i_2} = \frac{v_2}{v_1} \Rightarrow v_1\, i_1 = v_2\, i_2 \qquad \ldots$$

(4.10)

(Instantaneous power into primary) = (Instantaneous power out of secondary)

In terms of rms values,

i.e. VA output = VA input, i.e. $V_1\, I_1 = V_2\, I_2$

Since $$\frac{V_1}{V_2} = \frac{N_1}{N_2},$$

So, $$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad \ldots$$

(4.11)

The circuit representation of Figure 4.4, can be simplified by referring the load impedance and secondary current to the primary side. From Figure 4.4, we see that

$$V_2 = I_2\, Z_L$$

or $$V_1 \cdot \frac{N_2}{N_1} = I_1 \cdot \frac{N_1}{N_2} \cdot Z_L$$

or $$V_1 = I_1 \left(\frac{N_1}{N_2}\right)^2 Z_L = I_1 \cdot Z_L' \qquad \ldots$$

(4.12)

where $Z_L' = \left(\dfrac{N_1}{N_2}\right)^2 Z_L$ is said to be the load impedance referred to the primary side.

From $V_2 = I_2 Z_L$ we can also easily obtain $V_2' = I_2' Z_L'$, where $V_2' = V_2 \left(\dfrac{N_1}{N_2}\right)$ is secondary terminal voltage referred to primary side, and $I_2' = I_2 \left(\dfrac{N_2}{N_1}\right)$ is secondary current referred to primary side. In the ideal transformer, $I_1 = I_2'$ and $V_1 = V_2'$.

4.3.3 Equivalent Circuit of a Real Transformer

In real conditions, in addition to the mutual flux which links both the primary and secondary windings transformer, has a component of flux, which links either the primary winding or the secondary, but not both. This component is called leakage flux. The flux which links only with primary is called primary leakage flux, and the flux which links only with secondary is called secondary leakage flux. Figure 4.4 shows schematically the mutual and the leakage flux.

From our knowledge of magnetic circuits, we know that a flux lining with a winding is the cause of inductance of the winding (Inductance = Flux linkage per ampere). Since in a transformer the flux is alternating, its flux linkage gives rise to an induced voltage in the winding. Thus, primary leakage flux (which is proportional to I_1) causes an induced voltage, which acts as a voltage drop. Similarly for the secondary leakage flux. The effect of these induced EMFs are, therefore, represented as inductive leakage reactances X_{l1}, X_{l2}.

X_{l1} and X_{l2} are called leakage reactances of the primary and secondary respectively. These are also denoted as X_1, X_2.

The windings of the transformer have resistance R_1, R_2. These resistances cause a voltage drop $I_1 R_1$ and $I_2 R_2$, as also ohmic loss $I_1^2 R_1$ and $I_2^2 R_2$.

To sum up, in a Real Transformer,

(a) Both the primary and secondary windings possesses resistance. As a result, the value of actual impressed voltage across the transformer is the voltage $\overline{V}_1$ less the drop across the resistance R_1. Moreover, the copper loss in the primary winding is $(I_1^2\ R_1)$ and in the secondary winding $(I_2^2\ R_2)$.

(b) A Real Transformer has some leakage flux, as shown in the Figure 4.4. These fluxes, as discussed earlier, lead to self-reactances X_{l_1} and X_{l_2} for the two windings respectively.

(c) The magnetizing current cannot be zero. Its value is determined by the mutual flux ϕ_m. The mutual flux also determines core-loss taking place in the iron-parts of the transformer. The value of I_o does not depend on load and hence the iron-loss or core-loss is constant which is not zero.

Considering the effects of resistances and leakage reactances, a transformer can be visualized as shown in Figure 4.5.

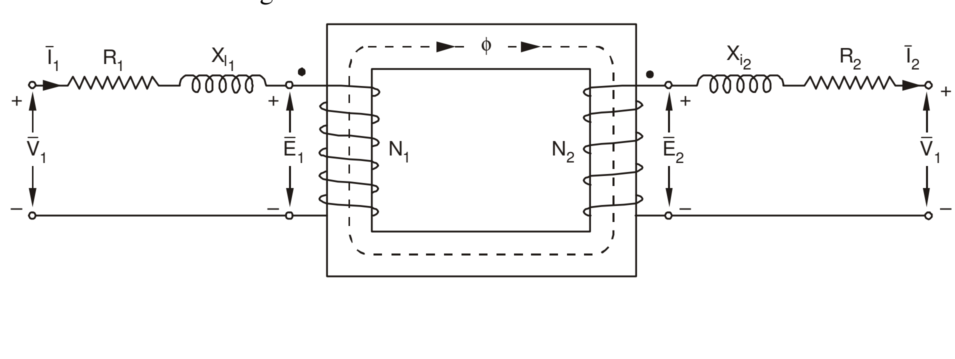


Figure 4.5 : Representation of Transformer Showing Leakage Reactances

In the form of equivalent circuit, this can be represented as in Figure 4.6.

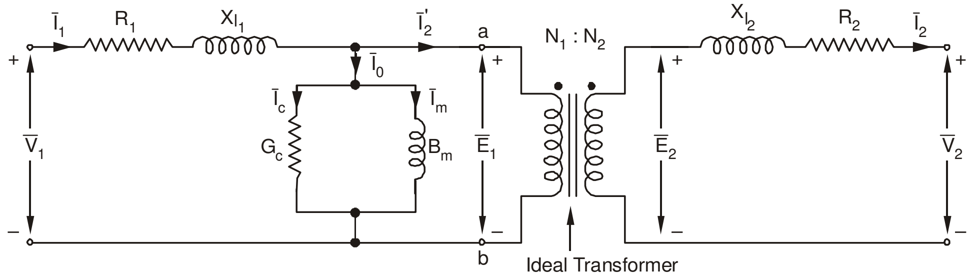


Figure 4.6 : Exact Equivalent Circuit of Real Transformer

The use of this equivalent circuit is difficult and calculations involved are laborious. For most practical purposes (like calculations of voltage regulation and efficiency) we need only a simplified form of equivalent circuit. We will now proceed to first obtain a **simplified** equivalent circuit and then to obtain an **approximate** equivalent circuit.

Equivalent Circuit Referred to Primary Side

We will now refer the impedance $R_2 + j\,X_{l_2}$ to the primary side i.e. to the left hand side of the ideal transformer. We have seen earlier that a load impedance Z_L can be referred to primary side as Z'_L, where

$$Z'_L = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Similarly $Z_2 = R_2 + j\,X_{l_2}$ can be referred to the primary side as

$$Z'_2 = \left(\frac{N_1}{N_2}\right)^2 Z_2 \qquad \ldots$$

(4.13)

where Z'_2 is said to be the secondary winding impedance referred to the primary side.

Eq. (4.13) can be re-written as

$$R'_2 + j\,X'_{l_2} = \left(\frac{N_1}{N_2}\right)^2 . (R_2 + j\,X_{l_2})$$

Equating real and imaginary parts

$$R'_2 = \left(\frac{N_1}{N_2}\right)^2 R_2$$

and
$$X'_{l_2} = \left(\frac{N_1}{N_2}\right)^2 X_{l_2} \qquad \ldots$$

(4.14)

R'_2 is the secondary winding resistance referred to primary, and X'_{l_2} is the secondary winding leakage reactance referred to primary side.

Figure 4.6 can now be modified (i.e. referring the secondary resistance and reactance to the primary side) to get the equivalent circuit shown in Figure 4.7.

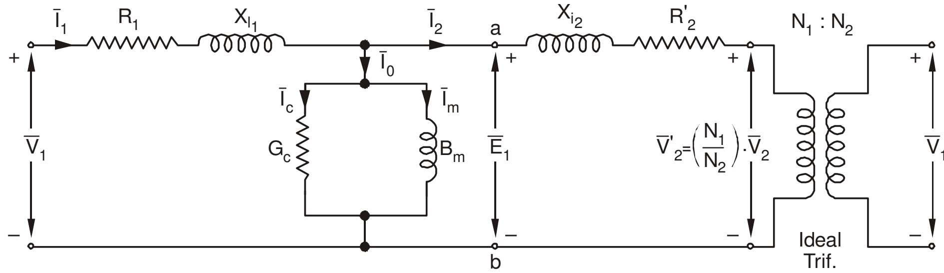


Figure 4.7 : Exact Circuit with Secondary Parameters Referred to Primary Side

The secondary terminal voltage V_2 and secondary current I_2 can also be referred to the primary side using the relations.

$$\frac{V_2'}{V_2} = \frac{N_1}{N_2}$$

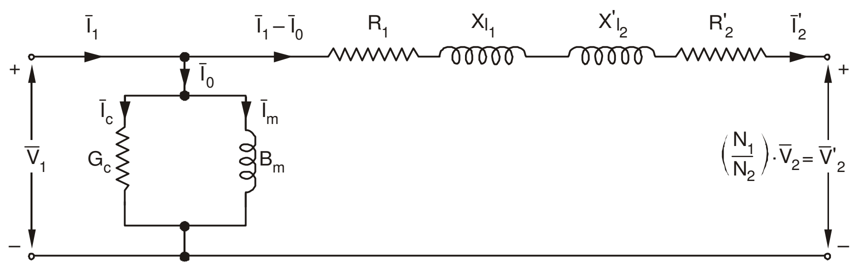
and
$$\frac{I_2'}{I_2} = \frac{N_2}{N_1}$$

These referred quantities $\bar{V}_2'$ and I_2' are also marked in Figure 4.7.

4.3.4 Approximate Equivalent Circuit

Transformers which are used at a constant power frequency (say 50 Hz), can have very simplified approximate equivalent circuits, without having a substantial effect on the performance evaluation (efficiency and voltage regulation). It should be borne in mind that 'higher the VA or KVA rating of the transformers, better are the approximation-based evaluation results.'

It is assumed that $\bar{V}_1 \approx \bar{E}_1$ ($\bar{V}_1$ is approximately equal to $\bar{E}_1$) even under conditions of load. This assumption is justified because the values of winding resistance and leakage reactances are very small. Therefore, the exciting current drawn by the parallel combination of conductance G_c and susceptance B_m would not be affected significantly by shifting it to the input terminals. With this change, the equivalent circuit becomes as shown in Figure 4.8(a).

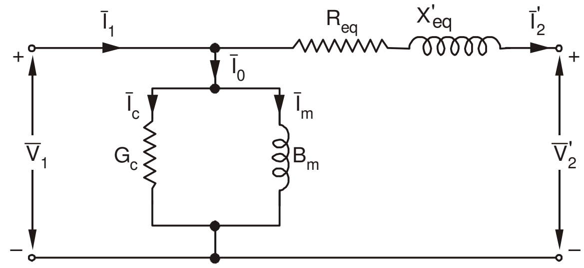


(a) Equivalent Circuit Referred to Primary Side

Denoting
$$R_1 + R_2' = R'_{\text{eq}}$$

and
$$X_{l_1} + X_{l_2}' = X_{\text{eq}}'$$

The equivalent circuit becomes as shown in Figure 4.8(b) R'_{eq}, X_{eq}' are called the equivalent resistance and equivalent reactance referred to primary side.



(b) Approximate Equivalent Circuit

Figure 4.8

If only voltage regulation is to be calculated even the excitation branch can be neglected and the equivalent circuit becomes as shown in Figure 4.9.

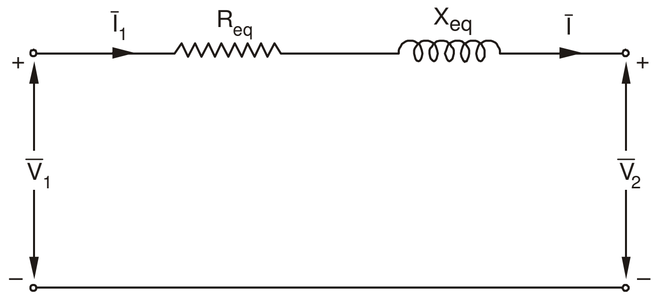


Figure 4.9 : Most Simplified Form of Approximate Equivalent Circuit

(Note that the equivalent circuit parameters, can be referred to the secondary side also.)

4.4 PHASOR DIAGRAM AND VOLTAGE REGULATION

The phasor diagram or vector diagram of a transformer for the no load case was discussed before. The phasor diagram for a loaded transformer depends on, whether the resistances and reactances of the primary and secondary winding have been considered or neglected.

We shall stick to some of the approximate equivalent circuit.

4.4.1 Phasor Diagram at Load without Winding Resistance and Reactance

The starting point of all phasor diagrams is the mutual flux phasor. The induced voltage in the two windings lag behind the flux phasor by 90^o. Now we will proceed to obtain the phasor diagram for three specific load power factors, viz.,

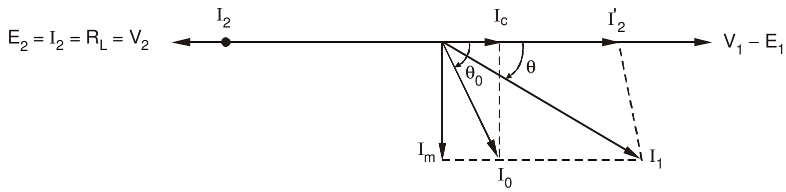
(a) pure resistive load

(b) inductive or lagging pf load, and

(c) capacitive or leading pf load.

Resistance Load

The phasor diagram neglecting winding resistance and reactance is given in Figure 4.10(a). E_1, E_2 lag behind ϕ_m by 90^o. The load current I_2 being at unity power factor is in phase with E_2. Corresponding to the load current the primary draws an additional current I'_2 (in addition to no load current). The magnitude of I'_2 is $\left(\frac{N_2}{N_1}\right)$ times the magnitude of I_2. Phase position of I'_2 is opposite to that of I_2, so that the ampere turns of secondary and primary can balance each other.



(a) Phasor Diagram for Resistive Load (Neglecting Winding Resistance and Reactance)

The primary current I will be phasor sum of I_2' and no load current I_0.

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

and

$$\bar{I}_0 = \bar{I}_c + \bar{I}_m$$

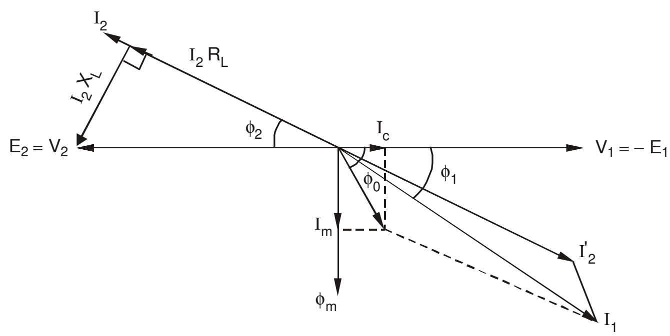
ϕ_0 : Phase angle of at no-load

ϕ_1 : Phase angle at load (between current I_1 and V_1).

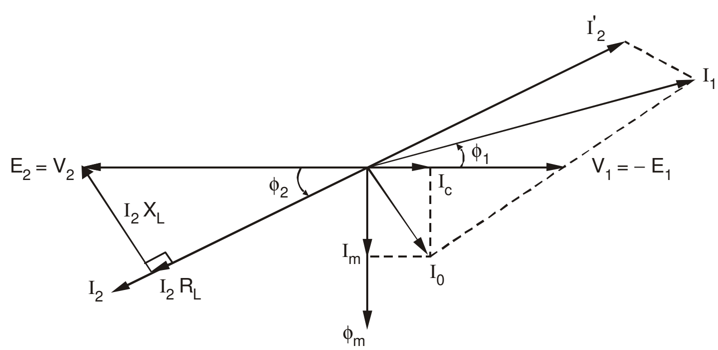
For Inductive Load

For an inductive load (i.e. $R_L + j\,X_L$), the load current (i.e. secondary winding current) I_2 will lag the secondary voltage E_2 by an angle ϕ_2. $\bar{I}_2'$ is in direct opposition to I_2 in the phasor diagram. The primary current I_1 is the phasor sum of I_0 and $\bar{I}_2'$. Once again ϕ_0 is the phase angle of the no load current and ϕ_1 is the phase angle of input current. The phasor diagram is shown in Figure 4.10(b).

Phasor diagram for a capacitive load (leading power factor), i.e. $R_L - j\,X_L$ can be similarly drawn, as shown in Figure 4.10(c).



(b) Phasor Diagram for Inductive Load (Neglecting Winding Resistance and Reactance)



(c) Phasor Diagram for Capacitive Load (Neglecting Winding Resistance and Reactance)

Figure 4.10

SAQ 2

The magnetizing current on the HT side of a 440/220 V single-phase transformer is 2.8 A. Determine the HT current and the power factor for the following loads on the LT side.

(i) 25 amperes at unity power factor.

(ii) 25 amperes at 0.85 power factor lagging.

Neglect iron-loss component of no-load current.

4.4.2 Phasor Diagram at Load with Winding Resistance and Reactance

Since the basics of phasor diagram with resistive, inductive and capacitive loads have already been considered in Figures 4.10(a), (b) and (c) respectively, we now restrict ourselves to the more commonly occurring load i.e. inductive along with resistance, which has a lagging power factor.

For drawing this diagram, we must remember that

$$\overline{V_2} = \overline{E_2} - \overline{I_2}\ (R_2 + j\ X_{l2})$$

and

$$\overline{V_1} = -\ \overline{E_1} + \overline{I_2}\ (R_1 + j\ X_{l1})$$

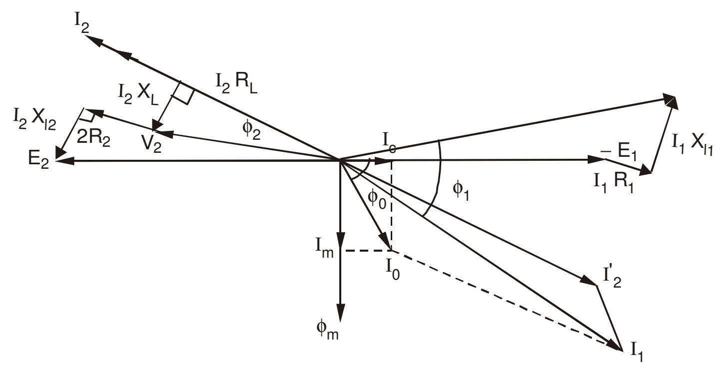


Figure 4.11 : Complete Phasor Diagram of a Transformer (for Inductive Load or Lagging pf)

(**Note :** Phasor diagram can also be drawn, using various approximations.)

4.4.3 Voltage Regulation

Under no load conditions, the voltage at the secondary terminals is E_2 and $E_2 \approx V_1 \cdot \frac{N_2}{N_1}$

(This approximation neglects the drop R_1 and X_{l1} due to small no load current). As load is applied to the transformer, the load current or the secondary current increases. Correspondingly, the primary current I_1 also increases. Due to these currents, there is a voltage drop in the primary and secondary leakage reactances, and as a consequence the voltage across the output terminals or the load terminals changes. In quantitative terms this change in terminal voltage is called Voltage Regulation.

Voltage regulation of a transformer is defined as the drop in the magnitude of load voltage (or secondary terminal voltage) when load current changes from zero to full load value. This is expressed as a fraction of secondary rated voltage

$$\text{Regulation} = \frac{\text{Secondary terminal voltage at no load} - \text{Secondary terminal voltage at any load}}{\text{Secondary rated voltage}}$$

The secondary rated voltage of a transformer is equal to the secondary terminal voltage at no load (i.e. E_2), this is as per IS.

Voltage regulation is generally expressed as a percentage.

Percent voltage regulation (% VR) $= \dfrac{E_2 - V_2}{E_2} \times 100$.

Note that E_2, V_2 are magnitudes, and not phasor or complex quantities. Also note that voltage regulation depends not only on load current, but also on its power factor.

Using approximate equivalent circuit referred to primary or secondary, we can obtain the voltage regulation.

From approximate equivalent circuit referred to the secondary side and phasor diagram for the circuit.

$$E_2 = V_2 + I_2\ r_{eq}\ \cos\ \phi_2 \pm I_2\ x_{eq}\ \sin\ \phi_2$$

where $r_{eq} = r_2 + r_1^1$ (referred to secondary) $x_e = x_2 + x_1^1$ (+ sign applies lagging power factor load and – sign applies to leading pf load).

So
$$\frac{E_2 - V_2}{E_2} = \frac{I_2\ r_{eq}\ \cos\ \phi_2 \pm I_2\ x_{eq}\ \sin\ \phi_2}{E_2}$$

$$\frac{E_2 - V_2}{E_2} = \frac{I_2\ r_{eq}}{E_2}\cos\ \phi_2 \pm \frac{I_2\ x_{eq}}{E_2}\sin\ \phi_2$$

% Voltage regulation = (% resistive drop) cos ϕ_2 ± (% reactive drop) sin ϕ_2.

Ideally voltage regulation should be zero.

SAQ 3

A single-phase transformers has 2% resistive drops and 5% reactive drop. Calculate its VR at (a) 0.8 lagging PF and (b) 0.8 leading PF.

4.5 LOSSES AND EFFICIENCY OF TRANSFORMER

A transformer does't contain any rotating part so it is free from friction and windage losses. In transformer the losses occur in iron parts as well as in copper coils. In iron core the losses are sum of hysteresis and eddy current losses. The hysteresis losses are $P_h \propto f\, B^x_{max}$ and eddy current loss is equal to $P_e \propto f^2\, B_{max}$.

Where "f" is frequency "B_{max}" is maximum flux density.

We know that the maximum flux density is directly proportional to applied voltage so if the applied voltage is constant then the flux density is constant and the hysterises losses are proportional to f and eddy current losses are proportional to f^2.

4.5.1 Iron Losses or Core Losses

To minimize hysteresis loss in transformer, we use Cold Rolled Grain Oriented (CRGO) silicon steel to build up the iron core.

Eddy Current Loss

When the primary winding variable flux links with iron core then it induces some EMF on the surface of core. The magnitude of EMF is different at various points in

core. So, there is current between different points in Iron Core having unequal potential.

These currents are known at eddy currents. $I^2 R$ loss in iron core is known as eddy current loss. These losses depends on thickness of core. To minimize the eddy current losses we use the Iron Core which is made of laminated sheet stampings. The thickness of stamping is around 0.5 mm.

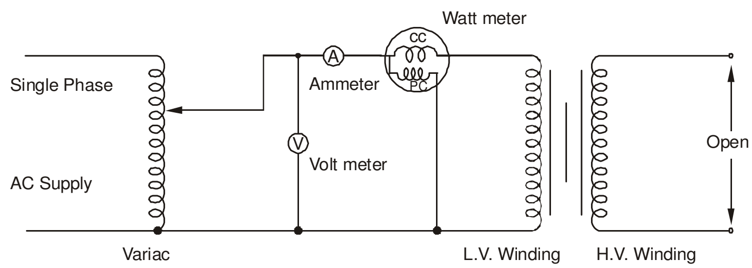
Determination of Iron or Core Losses

Practically we can determine the iron losses by performing the open circuit test.

Open Circuit Test

We perform open circuit test in low voltage winding in transformer keeping the high voltage winding open. The circuit is connected as shown in Figure 4.12(a). The instruments are connected on the LV side. The advantage of performing the test from LV side is that the test can be performed at rated voltage.

When we apply rated voltage then watt meter shows iron losses [There is some copper loss but this is negligible when compared to iron loss]. The ammeter shows no load current I_0 which is very small [2-5 % of rated current]. Thus, the drops in R_1 and X_{l1} can be neglected.



(a) Open Circuit Test

We have W_0 = iron loss

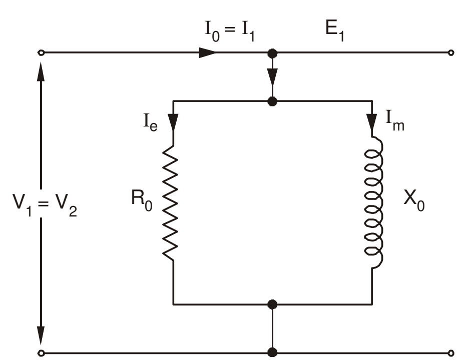
I_0 = no load current

then $$\cos \phi = \frac{W_0}{V_i \; I_0}$$

So $$I_e = I_0 \cos \phi$$

and $$I_m = I_0 \sin \phi .$$

Under no load conditions the PF is very low (near to 0) in lagging region. By using the above data we can draw the equivalent parameter shown in Figure 4.12(b).



(b) No Load Equivalent Circuit from Open Circuit Test

Figure 4.12

where $$R_0 = \frac{V_1}{I_e}$$

and $$X_0 = \frac{V_1}{I_m}$$

Example 4.3

At no load test, a transformer draws a current of 0.2 Ampere lagging behind the applied voltage by 80°, when the low voltage winding of the transformer is connected to a 500 V source. Calculate (a) iron loss and (b) components of the no load current.

Solution

We have $V_1 = 500$ V, $I_o = 0.2$ A and $\phi_o = 80^\circ$

$$\cos 80^\circ = 0.1736$$

$$\text{PF} = \cos\phi \approx 0.174 \text{ lagging}$$

(a) Iron loss $= V_1 I_o \cos\phi$

$$= 500 \times 0.2 \times 0.174$$

$$= 17.4 \text{ watts}$$

(b) Components of no load current

$$I_c = I_o \cos\phi = 0.0348 \text{ A}$$

$$I_m = I_o \sin\phi = 0.197\ A \ \ or \ \ I_m = \sqrt{I_0^2 - I_c^2} = 0.197$$

4.5.2 Copper Losses

In a transformer the primary and secondary winding currents increase with increases in load. Due to these currents there is some $I^2 R$ losses. These are known as copper losses or ohmic losses. The total $I^2 R$ loss in both windings at rated or full load current is equal to $I_1^2 R_1 + I_2^2 R_2$.

$$\text{Copper loss} = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_1 + \left[I_2' \left(\frac{N_1}{N_2} \right) \right]^2 R_2 = I_1^2 R_1 + I_1'^2 R_2' = I_1^2 (R_1 + R_2')$$

[assuming $I_1' = I_1$, i.e. shunt branch neglected].

$$= I_1^2 R_{0_1}$$

Similarly, it can be shown that

$$\text{Copper loss} = I_2^2 R_{0_2}$$

Here I_1 and I_2 are primary and secondary current. R_1 is primary winding resistance and R_2 is secondary winding resistance.

R_{0_1} is total resistance of winding referred to primary R_{0_2} is total resistance of windings referred to secondary.

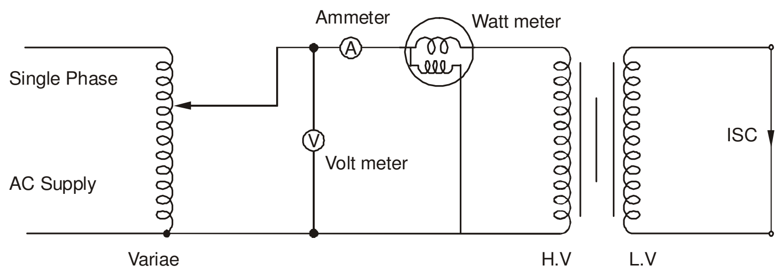
By performing short circuit test we find out copper loss experimentally.

Short Circuit Test

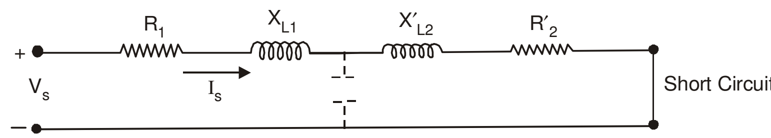
It's an indirect method to find out the copper losses. To perform this test, we apply a reduced voltage to the primary winding through instruments keeping LV winding short circuited. The connections are shown in Figure 4.13(a). We need to apply only 5-10% of rated voltage to primary to circulated rated current in the primary and

secondary winding. The applied voltage is adjusted so that the ammeter shows rated current of the winding. Under this condition, the watt-meter reading shows the copper losses of the transformer. Because of low value of applied voltage, iron losses, are very small and can be neglected.

As applied voltage is very small, small voltage across the excitation branch produces very small percentage of exciting current in comparison to its full load current and can therefore, be safely ignored. As a result, equivalent circuit with secondary short circuited can be represented as Figure 4.13(b).



(a) Short Circuit Test



(b) Transformer Equivalent Circuit with Secondary Short Circuited

Figure 4.13

At a rated current watt meter shows full load copper loss. We have

V_s = applied voltage

I_s = rated current

W_s = copper loss

then, equivalent resistance $R_{eq} = \dfrac{W_s}{I_s^2} = R_1 + R_2'$

and equivalent impedance $Z_{eq} = \dfrac{V_s}{I_s}$

So we calculate equivalent reactance

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = X_{L1} + X_{L2}'$$

These R_{eq} and X_{eq} are equivalent resistance and reactance of both windings referred in HV side. These are know as equivalent circuit resistance and reactance.

4.5.3 Efficiency of Single Phase Transformer

Generally we define the efficiency of any machine as a ratio of output power to the input power, i.e.

$$\text{efficiency } (\eta) = \frac{\text{Output Power}}{\text{Input Power}} = \frac{\text{Input Power} - \text{losses}}{\text{Input Power}}$$

$$= 1 - \frac{\text{losses}}{\text{Input Power}}$$

Alternatively,
$$\eta = \frac{\text{Output Power}}{\text{Output Power} + \text{Losses}}$$

In a transformer, if P_i is the iron loss, and P_c is the copper loss at full load (when the load current is equal to the rated current of the transformer, the total losses in the transformer are $P_i + P_c$. In any transformer, copper losses are variable and iron losses are fixed.

When the load on the transformer is ($x \times$ full load), the copper loss will be total $x^2 \; P_c$ and total losses $= P_i + x^2 \; P_c$.

P_c is full load copper loss and 'x' is the ratio of load current to the full load current. If the output power of the transformer is $x \; V_2 \; I_2 \cos \phi$, then efficiency ($\eta$) becomes,

$$\eta = \frac{x \; V_2 \; I_2 \; \cos \phi}{V_2 \; I_2 \; \cos \phi + P_i + x^2 \; P_c}$$

or
$$\eta = \frac{x \; KVA \text{ Rating} \times \cos \phi}{KVA \text{ Rating} \times \cos \phi + P_i + x^2 \; P_c}$$

The efficiency varies with load. So, we can find the condition under which the η is maximum. For maximum efficiency,

$$\frac{d\eta}{d I_2} = 0$$

or
$$\frac{d\eta}{d I_2} = \frac{d}{d I_2} \left(\frac{x \; V_2 \; I_2 \; \cos \phi}{x \; V_2 \; I_2 \; \cos \phi + P_i + x^2 \; P_c} \right) = 0$$

Solving this, we get $\quad P_i = x^2 \; P_c$

or $\quad$ iron loss = copper loss

The copper loss varies with load current I_2 so when the copper losses are equal to the iron losses for a particular load then efficiency (η) of the transformer is maximum. This is called condition for maximum efficiency.

The maximum efficiency $(\eta_{\max.}) = \dfrac{x \, V_2 \; I_2 \; \cos \phi}{x \, V_2 \; I_2 \; \cos \phi + 2P_i}$

Now, we determine the load at which the maximum efficiency occurs.

From the condition of maximum efficiency, we have

$$P_i = x^2 \; P_c$$

on
$$x = \sqrt{\frac{P_i}{P_c}}$$

Thus, the load at which efficiency is maximum occurs, is given by

$$x \text{ (Rated } KVA) = \sqrt{\frac{P_i}{P_c}} \text{ (Rated kVA)}$$

Example 4.4

A 10 KVA transformer has 400 watt iron losses and 600 watt copper losses. Determine maximum efficiency of the transformer at 0.8 power factor lagging. Also calculate the load at which the $\eta_{\max.}$ occurs.

Solution

We know that for efficiency to be maximum

Now
$$x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{400}{600}} = \sqrt{\frac{2}{3}} = 0.8165$$

Then load KVA at which the $\eta_{\max.}$ occurs, i.e. output

$$= \sqrt{\frac{2}{3}} \times 10 \text{ kVA}$$

$$= 0.8165 \times 10 \text{ kVA} = 8.165 \text{ kVA}$$

Copper loss = Iron loss

Then total losses = 2 × iron loss

$$= 2 \times 400$$

$$= 800 \text{ watt} = 0.8 \text{ kW};$$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output + losses}}$$

Now $$\eta_{max.} = \frac{x \text{ kVA} \times \cos \phi}{x \text{ kVA} \times \cos \phi + \text{losses}}$$

$$= \frac{0.8167 \times 10 \times 0.8}{0.1867 \times 10 \times 0.8 + 0.8} = 0.891$$

SAQ 4

(a) A single-phase 100/200 V, 1 kVA transformer has copper losses in h. v. side at 5 A equal to 80 W and iron losses as 60 W. Find the efficiency of the transformer at full load upf, and half load upf.

(b) Calculate full load efficiency at 0.8 power factor, for a 4 kVA, 200/400 V, 50 Hz, single-phase transformer with the following test results :

Open circuit test (LT side data) = 200 V, 0.8 Ampere, 70 Watts

Short circuit test (HT side data) = 17.5 V, 9 Ampere, 50 Watts

(c) A 440/110 V, 100 kVA, single-phase transformer has iron losses of 1.4 kW and a full load copper loss of 1.7 kW. Determine the kVA load for maximum efficiency. Also find the efficiency at half load 0.8 pf lagging.

(d) The following data were obtained on a 50 kVA 2400/120 V transformer :

Open circuit test, instruments on low voltage side :

Waltmeter reading = 396 Watts

Ammeter reading = 9.65 amperes

Voltmeter reading = 120 Volts

Short circuit test, instruments on high voltage side :

Waltmeter reading = 810 Watts

Ammeter reading = 20.8 amperes

Voltmeter reading = 92 Volts

Find the efficiency when rated kVA is delivered to a load having a power factor of 0.8 lagging.

4.5.4 All Day Efficiency (Energy Efficiency)

In electrical power system, we are interested to find out the all day efficiency of any transformer because the load at transformer is varying in the different time duration of the day. So all day efficiency is defined as the ratio of total energy output of transformer to the total energy input in 24 hours.

$$\text{All day efficiency} = \frac{\text{kWh output during a day}}{\text{kWh input during the day}}$$

here kWh is kilowatt hour.

Example 4.5

For a 100 KVA 10 KV/500 V transformer the electrical loading for different time durations in a day is shown below in Table 4.1.

Table 4.1

Time Duration	Load	PF
4 hrs	Full load	0.9 lagging
6 hrs	½ load	Unity PF
4 hrs	Full load	0.8 PF lagging
10 hrs	No load	-

Determine the all day efficiency of the transformer, if the iron losses are 2 kW and full load copper losses are 4 kW.

Solution

For a transformer the iron loss are fixed total energy loss in iron loss, over 24 hours $= 2 \times 24 = 48$ kWh .

Energy loss due to copper loss at full load for 8 hours = (4 × 8) = 32 kWh.

Energy loss at half load due to copper losses

$$= 6 \times \left(\frac{1}{2}\right)^2 \times 4$$

$$= 6 \times \frac{1}{4} \times 4 = 6 \text{ kWh}$$

At no load there is no copper loss.

$\therefore$ The total energy loss due to copper loss in 24 hours

= (32 + 6) kWh = 38 kWh.

Total energy losses = Iron losses + Copper losses

= 48 + 38

= 86 kWh.

kWh Output for Transformer

At full load 0.9 lagging PF for 4 hrs kWh output

$= 100 \times 0.9 \times 4 = 360$ kWh

At half load unity PF for 6 hrs, kWh output

= 1/2 × 100 × 1 × 6 = 300 kWh

For full load at 0.8 PF for 4 hours

$= 100 \times 0.8 \times 4 = 320$ kWh

Now for rest 10 hr output is zero. So total kWh output in all day

= 360 + 300+ 320 = 980 kWh.

So all day efficiency $(\eta) = \dfrac{980}{980 + 86} = 0.9193 \text{ or } 91.9\%$.

4.6 SPECIAL TRANSFORMERS

In order to meet various industrial applications, many transformers are constructed with special design features. In this section you will be introduced to :

(a) autotransformers, and

(b) instrument transformers.

4.6.1 Auto-transformers

The transformers we have considered so far are two-winding transformers in which the electrical circuit connected to the primary is electrically isolated from that connected to the secondary. An auto-transformer does not provide such isolation, but has economy of cost combined with increased efficiency. Figure 4.24 illustrates the auto-transformer which consists of a coil of N_A turns between terminals 1 and 2, with a third terminal 3 provided after N_B turns. If we neglect coil resistances and leakage fluxes, the flux linkages of the coil between 1 and 2 equals $N_A\ \phi_m$ while the portion of coil between 3 and 2 has a flux linkage $N_B\ \phi_m$. If the induced voltages are designated as $\overline{E}_A$ and $\overline{E}_B$, just as in a two-winding transformer,

$$\frac{\overline{E}_A}{\overline{E}_B} = \frac{N_A}{N_B}$$

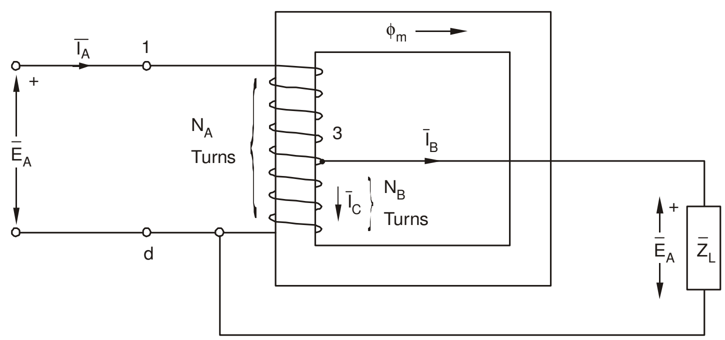


Figure 4.14 : Auto-transformer

Neglecting the magnetizing ampere-turns needed by the core for producing flux, as in an ideal transformer, the current $\overline{I}_A$ flows through only $(N_A - N_B)$ turns. If the load current is $\overline{I}_B$, as shown by Kirchhoff's current law, the current $\overline{I}_C$ flowing from terminal 3 to terminal 2 is $(\overline{I}_A - \overline{I}_B)$. This current flows through N_B turns. So, the requirement of a nett value of zero ampere-turns across the core demands that

$$(N_A - N_B)\ \overline{I}_A + (\overline{I}_A - \overline{I}_B)\ N_B = 0$$

or $$N_A\ \overline{I}_A - N_B\ \overline{I}_B = 0$$

Hence, just as in a two-winding transformer,

$$\frac{\overline{I}_A}{\overline{I}_B} = \frac{N_B}{N_A}$$

Consequently, as far as voltage, current converting properties are concerned, the autotransformer of Figure 4.14 behaves just like a two-winding transformer. However, in the autotransformer we don't need two separate coils, each designed to carry full load values of current.

4.6.2 Instrument Transformers

The generation and transmission parts of an electrical power system operate at voltages ranging from tens to hundreds of kilovolts and currents ranging from tens to hundreds of

Amperes. Despite this, such voltages are measured using voltmeters whose range is typically 0 – 150 V and ammeters whose range is 0 – 5 A. This is achieved by the use of **instrument transformers**, which are of two types namely **potential transformers** and **current transformers**. If a potential transformer is used to measure the voltage of a high voltage system, the primary is connected across the voltage to be measured while a low range voltmeter is connected across (see Figure 4.15(a)) the secondary. Similarly, for the measurement of current, the current to be measured passes through the primary of a current transformer, and a low-range ammeter is connected across the secondary.

(a) Potential Transformer Connection **(b) Current Transformer Connection**

Figure 4.15

A potential transformer is a high precision transformer specially designed to maintain a constant ratio between primary and secondary terminal voltages and ensure that these voltages are almost exactly in phase. A current transformer is designed to function in a similar manner as regards primary and secondary currents.

4.7 TRANSFORMERS IN THREE PHASE SYSTEMS

For a proper understanding of this section you will need to revise your knowledge of balanced three phase systems. In particular, you should know

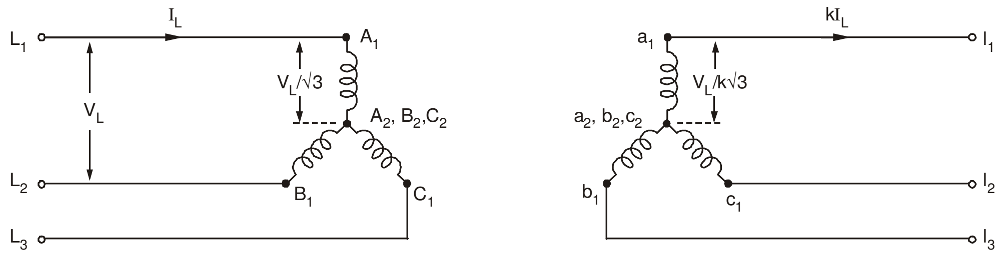
(a) the relations between line and phase quantities in star connected and delta connected balanced three phase circuits;

(b) expressions for three phase power and volt-amperes; and

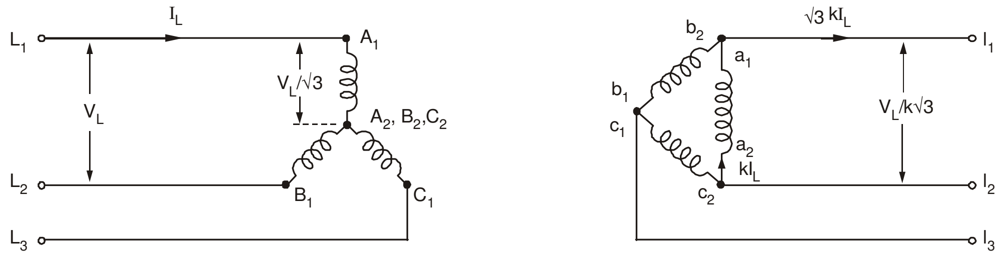
(c) equivalence relations between star-connected and delta-connected balanced systems.

4.7.1 Three-phase Bank of Single-phase Transformers

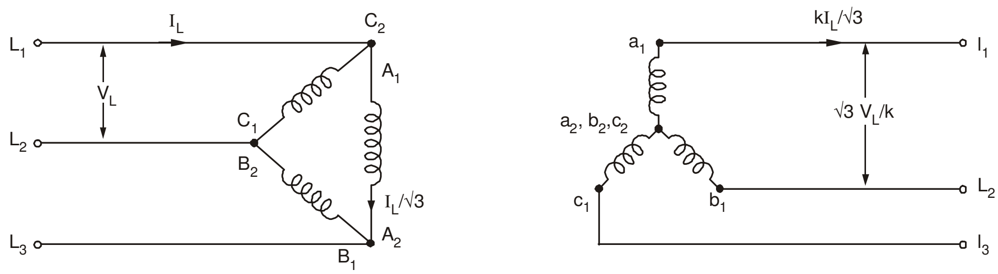
Electric power is generated, transmitted and distributed in three-phase form. Even where single-phase power is required, as in homes and small establishments, these are merely tapped off from a basic three-phase system. Transformers are, therefore, required to interconnect three phase systems at different voltage levels. This can be done using three single-phase transformers, constituting what is often called a transformer bank. The primary windings of three identical single-phase transformers can usually be connected either in star or in delta to form a three-phase system. Similarly, the secondary windings can also be connected in star or delta. We have, therefore, four methods of interconnection of primary/secondary, viz., star/star, star/delta, delta/star and delta/delta.



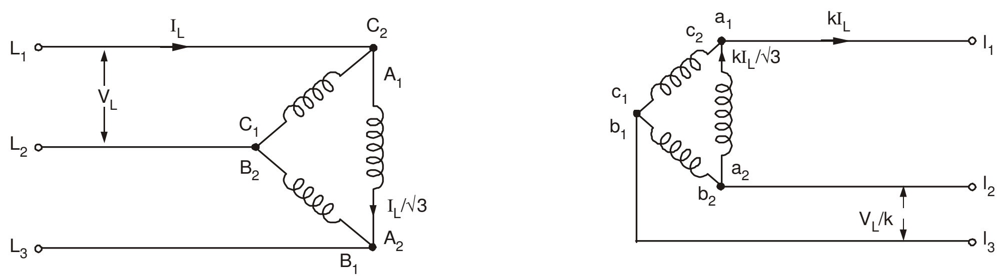
(a) Star/Star Connection (*Y* – *Y*)



(b) Star/Delta Connection (*Y* – Δ)



(c) Delta/Star Connection (Δ – *Y*)



(d) Delta/Delta Connection (Δ – Δ)

Figure 4.16 : Three Phase Transformer Connections

Let the primary to secondary turns ratio of each single-phase transformer be *k*. We will identify these transformers by the letters *A*, *B* and *C*. Transformers *A* will be assumed to have primary terminals A_1, A_2 and secondary terminals a_1, a_2, transformers *B* has terminals B_1, B_2 and b_1, b_2 and similarly for transformer *C*. We will also designate the three phase line terminals on the primary by *A*, *B*, *C* and the secondary line terminals by *a*, *b*, *c*. Further, we will suppose that in all the transformers the winding sense is such that on adopting a dot convention, dots would have to be marked next to primary and secondary terminals having the suffix 1. The four types of transformers connection would be as shown in Figure 4.16. The ratios of the primary and secondary line voltages is shown in this diagram, where *k* is the transformation ratio of one phase.

4.7.2 Three-phase Transformers

Instead of a bank of three separate single-phase transformers, each having its own separate iron-core, a single transformer can be designed to serve the same function. Such a single unit, called a three-phase transformer, has three primary windings and three secondary windings. These primary and secondary windings can be connected in star or in delta. The

connections and voltage relations of Figure 4.16 apply in this case also. Such a transformer differs from the single-phase transformers in the design of the iron-core. In the single-phase transformer bank the fluxes associated with a particular phase utilize an iron-core which serves only that phase, whereas in the three-phase transformer the iron-core couples different phases together. Because of this sharing of the iron-core by the three phases, such transformers can be built more economically. A three-phase transformer is always cheaper than *three* single-phase transformers used for the same purpose, weighs less and occupies less floor space.

Despite the above advantages, three single-phase transformers may be preferred if the conditions of operation are such that provision must be made for replacement. When using single-phase transformers it might be sufficient to provide just one single-phase transformer as a spare. If a three-phase transformer is used another three-phase transformer will be needed as a spare. While a three-phase transformer is cheaper than three single-phase transformers, it is much more expensive than one single-phase transformer. Secondly, there might exist situations like hydroelectric projects in remote locations, where it is not feasible to transport and install a heavy three-phase transformer and the use of three lighter single-phase transformers becomes the only feasible solution.

SAQ 5

It is proposed to transmit the power generated by a 200 MVA, 11 kV, 50 Hz, three-phase generator to a three-phase 220 kV transmission line using a bank of three single-phase transformers. Find the turns ratio and the voltage and current ratings for each single-phase transformer when the connections are

(a) Star/Star,

(b) Star/Delta,

(c) Delta/Star, and

(d) Delta/Delta.

4.8 SUMMARY

After going through this unit you should have understood the principles of operation of transformers and learnt the significance of mutual flux, leakage fluxes, core losses and winding resistances. You would be able to deduce the form of the exact equivalent circuit from theoretical considerations and determine the parameters of the approximate equivalent circuit from OC and SC tests. Based on this, you should be able to calculate efficiency and voltage regulation of a transformer on load. You would also have a general understanding of the use of transformers in power systems, including special transformers like auto-transformers, instrument transformers.

4.9 ANSWERS TO SAQs

SAQ 1

$$\phi_m = B_m \times A_i = 1.1 \times 150 \times 10^{-4} = 1.65 \times 10^{-2} \text{ Wb}$$

$$E_2 = 4.44\ f\ N_2\ \phi_m$$

$$= 4.44 \times 50 \times 66 \times 1.65 \times 10^{-2} = 24175.8 \times 10^{-2} = 241.76$$

$$I_2 = \frac{E_2}{Z_2}$$

$$I_2 = \frac{241.76}{4.0} = 60.44 \text{ A}$$

$$\text{Output } = E_2 \, I_2 = 241.76 \times 60.44 \text{ VA}$$

$$= 241.76 \times 60.44 \times 10^{-3} \text{ kVA} = 0.755 \text{ kVA}$$

$$= 14.61 \text{ kVA}$$

SAQ 2

Magnetizing current on the HT side = $-j$ 2.8 amp.

$$\text{Turns ratio } \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{440}{220} = 2$$

(a) Load current on LT side = $25 \angle -180^o$

Load current on HT side = $\frac{25}{2} \angle 0^o = 12.5 \angle 0^o$

$\therefore$ H. T. current $= 12.5 \angle 0 - j\, 2.8 = 12.81 \angle -12.6^o$

Power factor = cos ($-$ 12.6^o) = 0.976 lagging

(b) Load current on LT side $= 25 \angle (-\cos^{-1} 0.85 - 180^o)$

$$= 25 \angle (-31.8^o - 180^o) = 25 \angle -211.8^o$$

Load current on HT side $= \frac{1}{2} \times 25 \angle (-211.8^o + 180^o)$

$$= 12.5 \angle -31.8^o$$

$\therefore$ HT current $= 12.5 \angle -31.8^o - j\, 2.8$

$$= 10.623 - j\, 6.58 - j\, 2.8$$

$$= 10.623 - j\, 9.38$$

$$= 14.17 \angle -41.44^o$$

Power factor = cos ($-$ 41.44^o) = 0.75 lagging

SAQ 3

We know that

V. R. = % (Resistive drop) cos $\phi \pm$ % (Reactive drop) sin ϕ

(a) For 0.8 lagging P. F.

V. R. = (% Resistive drop) cos ϕ + (% Reactive drop) sin ϕ

$\because$ cos ϕ = 0.8 so sin $\phi = \sqrt{1 - (0.8)^2} = 0.6$

Now V. R. $= (2 \times 0.8) + (5 \times 0.6) = 1.6 + 3 = 4.6\%$

(b) For 0.8 leading P. F.

V. R. = (% Resistive drop) cos ϕ $-$ (% Reactive drop) sin ϕ.

$\because \cos\phi = 0.8$ so $\sin\phi = \sqrt{1-(0.8)^2} = 0.6$

Now V. R. $= (2\times 0.8) - (5\times 0.6) = 1.6 - 3 = -1.4\%$

SAQ 4

(a) $$\eta_x = \frac{x \text{ kVA} \times 1000 \times \cos\phi}{x \text{ kVA} \times 1000 \times \cos\phi + P_i + x^2\ P_c} \times 100$$

where x fraction of flux load. At full load

$$I_a = \frac{1000}{2000} = 5\text{ A (as given)}$$

$\therefore\ P_c = 80$ W, $\quad P_s = 60$ W

When $x = 1$, $pf = 1$,

So, $$\eta_{\text{full load}} = \frac{1\times 1000\times 1}{1000\times 1 + 60 + 80}\times 100 = 87.7\%$$

At half load; $x = ½$, $pf = 1$

$$\eta_{\text{half load}} = \frac{\frac{1}{2}\times 1\times 1000\times 1}{\frac{1}{2}\times 1000\times 1 + 60 + \frac{80}{4}}\times 100 = 86.20\%$$

(b) Full load secondary current $I_2 = \dfrac{4\times 10^3}{400} = 10$ amp.

Copper loss at 9 amp = 50 W

$\therefore$ Copper loss at 10 amp $= \left(\dfrac{10}{9}\right)^2 \times 50 = 61.73$ W

Iron loss = 70 W

Full load efficiency at 0.8 pf $= \dfrac{E_2\, I_2\ \cos\phi}{E_2\, I_2\ \cos\phi + W_i + W_c}$

$$\frac{400\times 10\times 0.8}{400\times 10\times 0.8 + 70 + 61.73} = 0.9605$$

$\therefore$ Efficiency = 96.05%

(c) Let the kVA load for maximum efficiency be x kVA

Iron loss at maximum efficiency = 1.4 kW

Copper loss at maximum efficiency $= \left(\dfrac{x}{100}\right)^2 \times 1.7$ kVA

At maximum efficiency, copper loss = iron loss

or $\left(\dfrac{x}{100}\right)^2 \times 1.7 = 1.4$ or $x = 90.75$ kW

At half load copper loss $= \left(\dfrac{50}{100}\right)^2 \times 1.7 = 0.425$ kW

$$\text{Efficiency at half load} = \left(\frac{50 \times 0.8}{(50 \times 0.8 + 0.425 + 1.4)}\right) \times 100$$

$$= \frac{40 \times 100}{41.825} = 95.6\%.$$

(d) At rated voltage, core loss = 396 W (from OC test)

$$\text{Rated current on h. v. side} = \frac{\text{kVA} \times 10^3}{V_1}$$

$$= \frac{50 \times 10^3}{2400}$$

or $I_1 = 20.833$ amp.

$\therefore$ Full load copper loss = 810 W

Total losses = 396 + 810 = 1206 W

Output = 50 kVA

$= 50 \times 10^3 \times 0.8 \quad (\because \cos\phi = 0.8)$

$= 40000$ W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output + losses}} \times 100$$

$$= \frac{40000 \times 100}{40000 + 1206} = 97.07\%$$

SAQ 5

Let subscripts 1 and 2 denote primary and secondary side with given data,

$$V_{L_1} = 11 \text{ kV}, V_{L_2} = 220 \text{ kV}, \frac{\text{MVA}}{\text{Phase}} = 66.67 \text{ MVA}$$

	V_{P_1} (kV)	V_{P_2} (kV)	$\frac{N_1}{N_2} = \frac{V_{P_1}}{V_{P_2}}$	$I_{P_1} = \frac{66.67}{V_{P_1}}$ (kA)	$I_{P_2} = \frac{66.67}{V_{P_2}}$ (kA)
Y/Y	$\frac{V_{L_1}}{\sqrt{3}} = 6.35$	$\frac{V_{L_2}}{\sqrt{3}} = 127$	1 : 20	10.49	0.52
Y/Δ	$\frac{V_{L_1}}{\sqrt{3}} = 6.35$	$V_{L_2} = 220$	1 : 34.6	10.49	0.30
Δ/Y	$V_{L_1} = 11$	$\frac{V_{L_2}}{\sqrt{3}} = 127$	1 : 11.5	6.06	0.52
Δ/Δ	$V_{L_1} = 11$	$V_{L_2} = 220$	1 : 20	6.06	0.30